[M2 QDCS-QMI] Tensor Computations
Tentative Syllabus

Course Description

This course explores the theory and applications of tensor computations, ranging from the basic operations on tensors to the numerical algorithms for the formation and manipulation of tensor decompositions and networks. After providing all theoretical fundamentals for the tensor algebra, the course focuses on the state-of-the-art research work that leverages tensor decompositions and networks in the domains of quantum computing, high performance computing, and AI/data analysis.

Prerequisites

- Linear Algebra (vector spaces, matrix-vector calculus, matrix decompositions - QR, SVD, Cholesky, LU)
- Basic programming in Python

Course Objectives

- Learn the mathematical foundation of tensors and tensor operations.
- Understand the philosophy of low-rank computations using tensor decompositions/networks.
- Implement tensor network computations using Python libraries such as NumPy or TT-toolbox.

- Explore the applications of tensor computations in in quantum computing, high performance computing, and AI/data science.

Topics Covered

Introduction to Tensor Algebra

- Definition and notation
- Scalars, vectors, matrices, and generalization to tensors
- Basic tensor operations: addition, multiplication, contraction
- Tensor products and outer products
- Einstein summation convention and index manipulation

Tensor Decompositions and Networks

- Refresher on matrix decompositions (QR, SVD, Cholesky, LU, low-rank decompositions)
- Introduction to tensor decompositions and networks
- Canonical polyadic decomposition (CPD)
- Tucker and Hierarchical Tucker decompositions
- Tensor-train decomposition (TT), Matrix product states (MPS) and Projected entangled-pair states (PEPS) networks

Numerical Methods for Tensor Computations

- Algorithms for computing tensor decompositions (Tensor SVD)
- Low-rank tensor arithmetic
- Tensor cross-approximation
- Optimization algorithms on tensor manifolds (ALS, AMEN)
- Tensor completion and recovery

Applications in Quantum Computing, High Performance Computing, and AI

- Computational challenges in tensor algorithms
- Tensor networks in quantum chemistry and physics, quantum simulation, ...
- Tensor decompositions in multivariate data analysis, neural networks, recommender systems, ...

Evaluation

- Research paper presentation: 100%

Resources

Supplementary reading:

- *Tensor Decompositions and Applications* by Tamara G. Kolda and Brett W. Bader.
- *Tensor Ranks for the Pedestrian for Dimension Reduction and Disentangling Interactions* by Alain Franc.
- *Tensor Spaces and Numerical Tensor Calculus* by Wolfgang Hackbusch.
- And many research papers and articles provided throughout the course.

Tools and Software

- Python with NumPy, SciPy, TT-toolbox